2.1A WKST

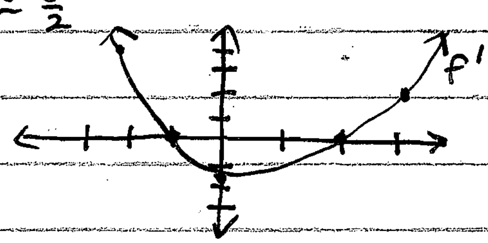
1a) ≈ 3

1b) ≈ 0

1c) $\approx -\frac{3}{2}$

1d) $\approx \frac{3}{2}$

2)



3) D 4) B 5) E 6) A 7) C

8) 6

9) 4

10) DNE, unbounded behavior

11) DNE b/c fractional exponent on bottom

12) DNE b/c $L \neq R$

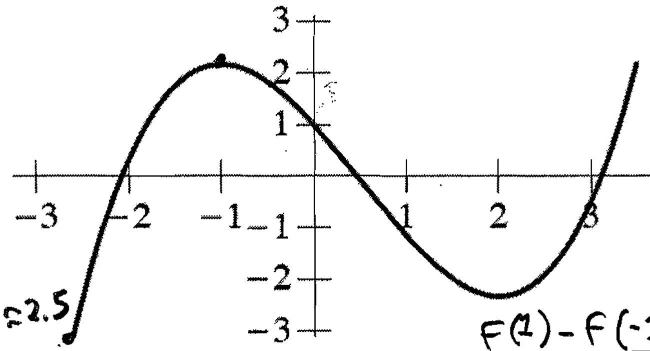
13) X is differentiable everywhere except at $x=3$ b/c it is a non-removable discontinuity

14) X is differentiable everywhere exept at $X=4$ due to a sharp turn/cusp is a jump discontinuity

15) X is differentiable at $(1, \infty)$. At $x=1$ is a Vertical Tangent.

16) X is differentiable exept at $X=0$. At $x=0$ is a jump discontinuity

The graph of a function f is given below.



$\frac{f'(-3)-f'(-1)}{-3+(+1)} = \frac{-3-2}{-2} = \frac{+5}{2} \approx 2.5$

$\frac{f(1)-f(-1)}{1--1} = \frac{-1-2}{2} = \frac{-3}{2}$

1) ESTIMATE the values of $f'(x)$ at each of the following x-values.

(a) $x = -2$ $\boxed{f'(-2) \approx 3}$

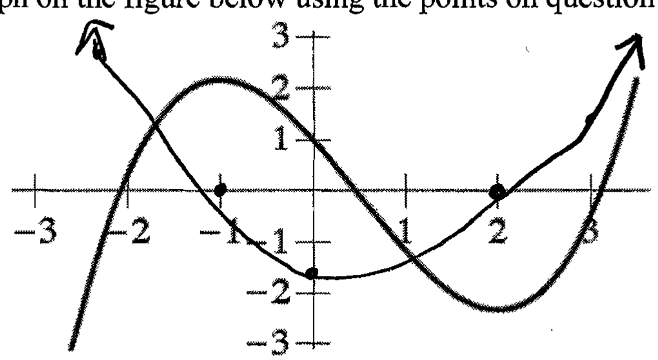
(b) $x = -1$ $\boxed{f'(-1) \approx 0}$

(c) $x = 0$ $\boxed{f'(0) \approx \frac{3}{-2}}$

(d) $x = 3$ $\boxed{f'(3) \approx \frac{3}{2}}$

2) Sketch the derivative graph on the figure below using the points on question 1.

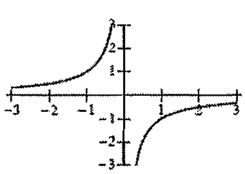
$\frac{f(4)-f(2)}{4-2} = \frac{3-}{2}$



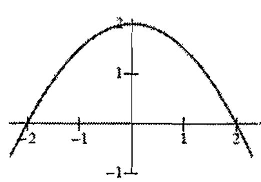
Match each graph of the function in the top row with the graph on the function's first derivative in the bottom row. Each choice will be used once. Write the letter in capital letters.

Function:

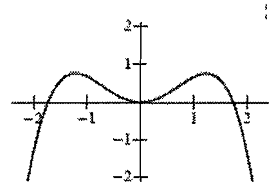
D 3)



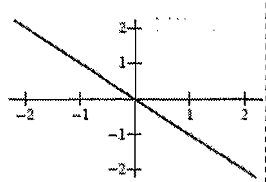
B 4)



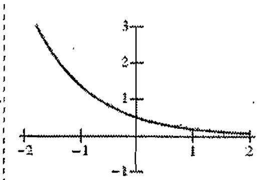
E 5)



A 6)

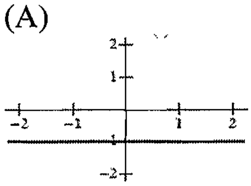


C 7)

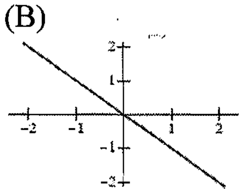


First Derivative:

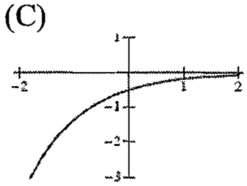
(A)



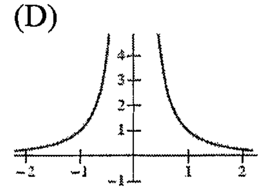
(B)



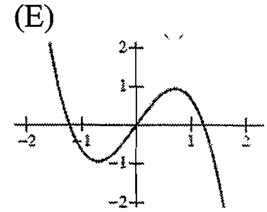
(C)



(D)



(E)



Use the alternative form of the derivative to find the derivative at $x = c$, if it exists.

8) $f(x) = x^2 - 5,\ c = 3$

9) $f(x) = x^3 + 2x^2 + 1,\ c = -2$

10) $g(x) = \sqrt{|x|},\ c = 0$

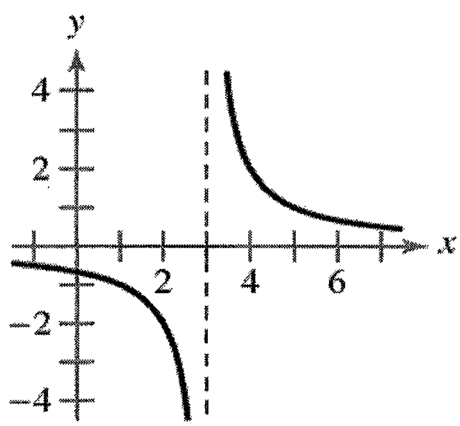
11) $f(x) = (x-6)^{2/3},\ c = -2$

12) $h(x) = |x+7|,\ c = -7$

Describe the x-values at which f is differentiable.

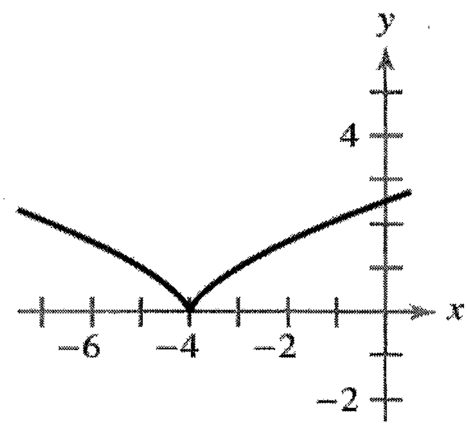
13)

$$f(x) = \frac{2}{x-3}$$



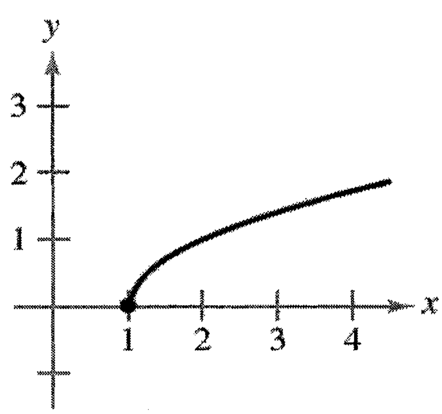
14)

$$f(x) = (x+4)^{2/3}$$



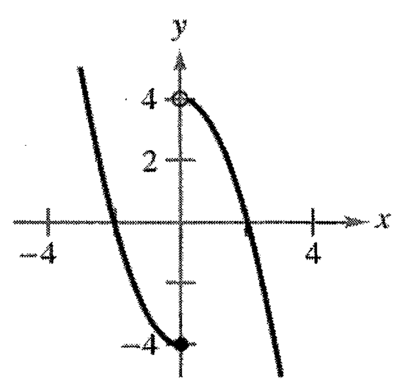
15)

$$f(x) = \sqrt{x-1}$$

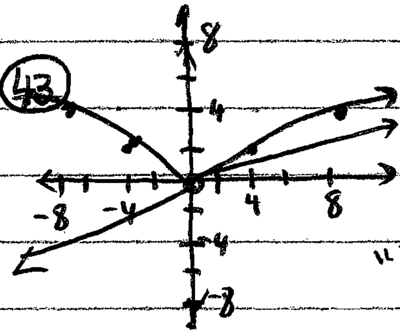


16)

$$f(x) = \begin{cases} x^2 - 4, & x \le 0 \\ 4 - x^2, & x > 0 \end{cases}$$



43

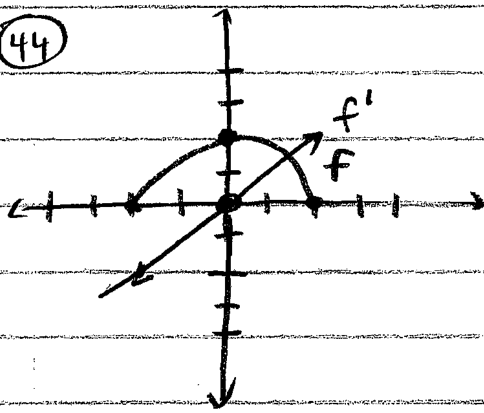


"DNE"

$$\frac{f(8)-f(4)}{8-4} = \frac{4-2}{4} = \frac{2}{4} = \frac{1}{2} = m$$

$$\frac{f(-8)-f(-4)}{-8-4} = \frac{4-2}{-4} = \frac{2}{-4} = \frac{-1}{2} = m$$

44



$$\frac{f(-2)-f(0)}{-2-0} = \frac{0-2}{-2} = +1$$

$$\frac{f(2)-f(0)}{2-0} = \frac{0-2}{2-0} = -1$$

47 Point $(4,5)$ + $(7,0)$ and $g(4)$ and $g'(4)$

$g(4) = 5$

$$g'(4) = \frac{g(4)-g(7)}{4-7} = \frac{5-0}{4-7} = \boxed{-\frac{5}{3} = m}$$

8 / 65 $f(x) = x^2 - 5,\ c = 3$

$$\lim_{x \to c} \frac{f(x)-f(c)}{x-c} \qquad \lim_{x \to 3} \frac{x^2-5-[3^2-5]}{x-3}$$

$$\lim_{x \to 3} \frac{x^2-5-9+5}{x-3} \qquad \lim_{x \to 3} \frac{x^2-9}{x-3} \qquad \lim$$

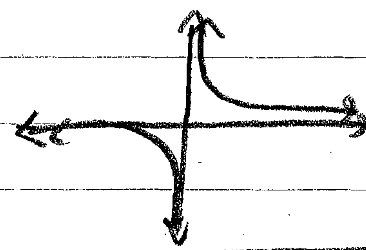
$$\lim_{x \to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} \qquad \lim_{x \to 3} x+3 = \boxed{6}$$

9/ 67 $f(x)=x^3+2x^2+1,\ c=-2$

$$=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}=\lim_{x\to -2}\frac{x^3+2x^2+1-[\cancel{-8}+\cancel{8}+1]}{x-(-2)}$$

$$=\lim_{x\to -2}\frac{x^3+2x^2+1-1}{x+2}=\lim_{x\to -2}\frac{x^3+2x^2}{x+2}=\lim_{x\to -2}\frac{x^2\cancel{(x+2)}}{\cancel{x+2}}$$

$$=\lim_{x\to -2}x^2=\boxed{4}$$



10/ 69 $g(x)=\sqrt{|x|}\ ;\ c=0$

$$\lim_{x\to c}\frac{f(x)-f(c)}{x-c}=\lim_{x\to 0}\frac{\sqrt{|x|}-\sqrt{0}}{x-0}=\boxed{\lim_{x\to 0}\frac{\sqrt{|x|}}{x}=DNE}$$

$$\lim_{x\to 0^-}\frac{\sqrt{|x|}}{x}=\frac{x^{1/2}}{x}=\frac{S}{B}=\pm\infty$$

$$\lim_{x\to 0^-}\frac{x^{1/2}}{x}=\quad -10\quad -100\quad -1000=\boxed{-\infty}$$

$$\lim_{x\to 0^+}\frac{x^{1/2}}{x}=\quad 10\quad 100\quad 1000\quad \boxed{+\infty}$$

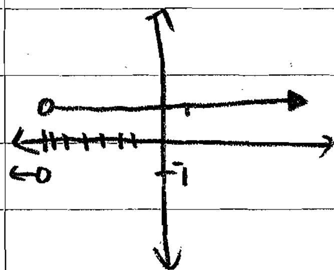
11/ 71 $f(x)=(x-6)^{2/3},\ c=6$

$$\lim_{x\to c}\frac{f(x)-f(c)}{x-c}=\lim_{x\to 6}\frac{(x-6)^{2/3}-\cancel{(6-6)^{2/3}}}{x-6}$$

$$\lim_{x\to 6}\frac{(x-6)^{2/3}}{(x-6)^{3/3}}=\lim_{x\to 6}\frac{1}{(x-6)^{1/3}}=$$

DNE b/c fractional exp on bottom

12/ 73) $h(x) = |x+7|$, $c = -7$ $\qquad \lim_{x\to -7} \frac{|x+7| - 0}{x+7} = \lim_{x\to -7} \frac{|x+7|}{x+7}$



Limit DNE b/c $L \neq R$.

$f(x) = \frac{2}{x-3}$

13/ 75) x is differentiable everywhere except at $x = 3$ b/c $x = 3$ is a non-removable discontinuity

14/ 77) $f(x) = (x+4)^{2/3}$

x is differentiable everywhere except at $x = -4$ due to a sharp turn, which is a jump discontinuity

15/ 79) $f(x) = \sqrt{x-1}$

x is differentiable from $(1, \infty)$. At $x = 1$, there is a vertical tangent

16) $f(x) = \begin{cases} x^2 - 4, & x \leq 0 \\ 4 - x^2, & x > 0 \end{cases}$

X is differentiable from $(-\infty, 0) \cup (0, \infty)$.

At $x = 0$ there is a jump discontinuity

9/67 $f(x)=x^3+2x^2+1$, $c=-2$

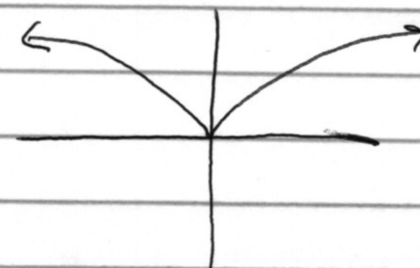
$$\lim_{x\to c}\frac{f(x)-f(c)}{x-c} = \lim_{x\to -2}\frac{x^3+2x^2-1-[-8+8+1]}{x-(-2)}$$

$$\lim_{x\to -2}\frac{x^3+2x^2+1-1}{x+2} = \lim_{x\to -2}\frac{x^3+2x^2}{x+2} = \lim_{x\to -2}\frac{x^2\cancel{(x+2)}}{\cancel{(x+2)}}$$

$$\lim_{x\to -2} x^2 = \boxed{4}$$

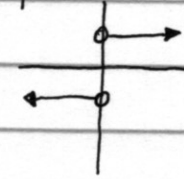
10/69 $g(x)=\sqrt{|x|}$; $c=0$

$$\lim_{x\to c}\frac{f(x)-f(c)}{x-c} \qquad \lim_{x\to 0}\frac{\sqrt{|x|}-\sqrt{0}}{x-0}$$



$$\lim_{x\to 0^-}\frac{\sqrt{|x|}-\sqrt{0}}{x-0} \qquad \lim_{x\to 0^+}\frac{\sqrt{|x|}-\sqrt{0}}{x-0}$$

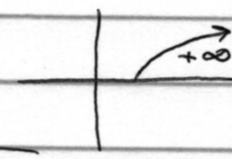
$-\frac{1}{1}, -\frac{3}{2}$ $\qquad$ $\frac{1}{1}, \frac{2}{2}$ -



11/71 $f(x)=(x-6)^{2/3}$, $c=6$

$$\lim_{x\to c}\frac{f(x)-f(c)}{x-c} = \lim_{x\to 6}\frac{(x-6)^{\frac{2}{3}}-(6-6)^{\frac{2}{3}}}{x-6}$$

$$\lim_{x\to 6}\frac{(x-6)^{2/3}}{(x-6)^{3/3}} = \lim_{x\to 6}\frac{1}{(x-6)^{1/3}} = \frac{1}{0}$$



DNE b/c f(x) is going to $+\infty$