Definition of a Derivative

Section 2.1

Calculus AP/Dual, Revised ©2017

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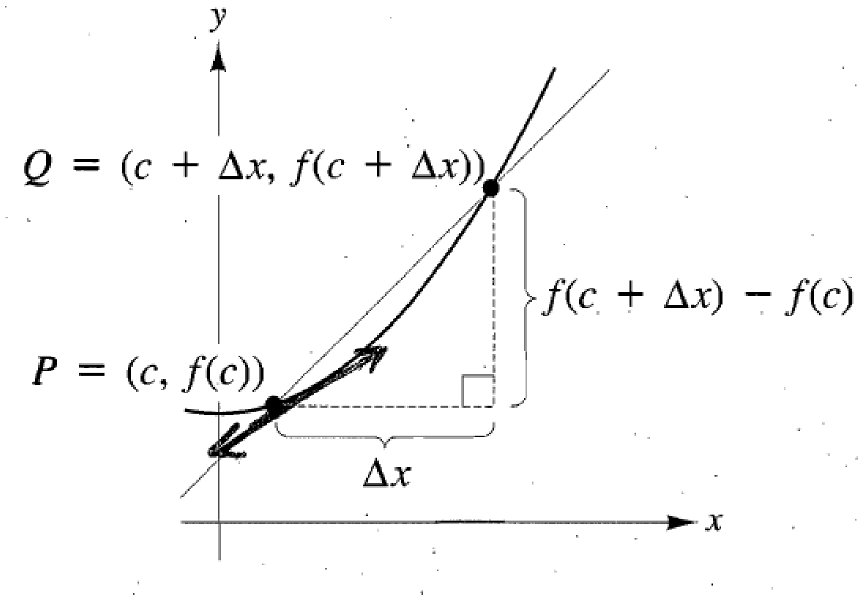
DEFINITION

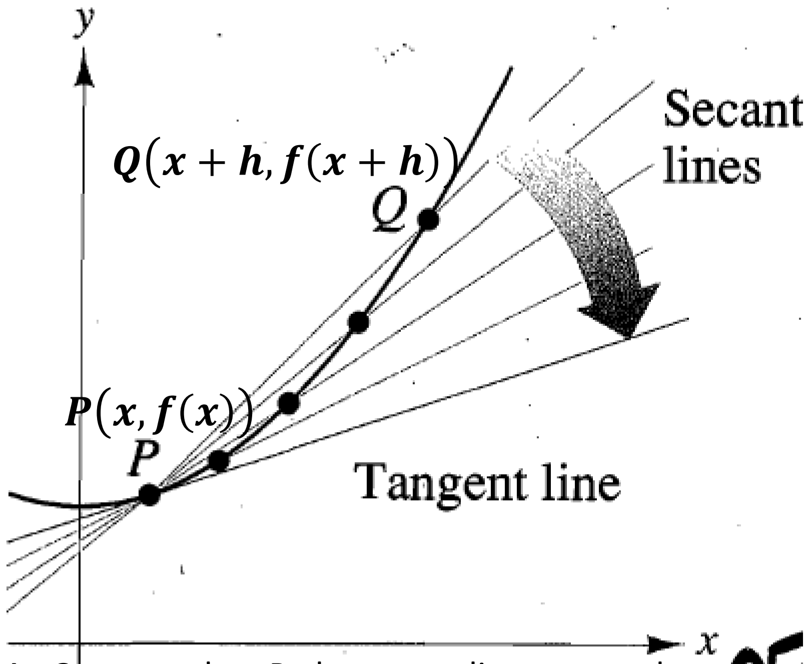
A. **The derivative of a function allows you to find the <u>SLOPE OF THE TANGENT LINE</u> at a point.**
   1. **The slope of a curve is <u>DIFFERENT</u> at every point whereas the slope of a line is the same.**

B. **Different ways of writing the derivative, it is known as "the derivative of y with respects to x"**
   1. $f'(x)$: **1st Derivative of f**
   2. $\frac{dy}{dx}$: **Derivative of y with respect to x**
   3. y': **1st Derivative of y**
   4. $\frac{d}{dx}[f(x)]$: **1st Derivative of $f(x)$ with respect to x**

EQUATION



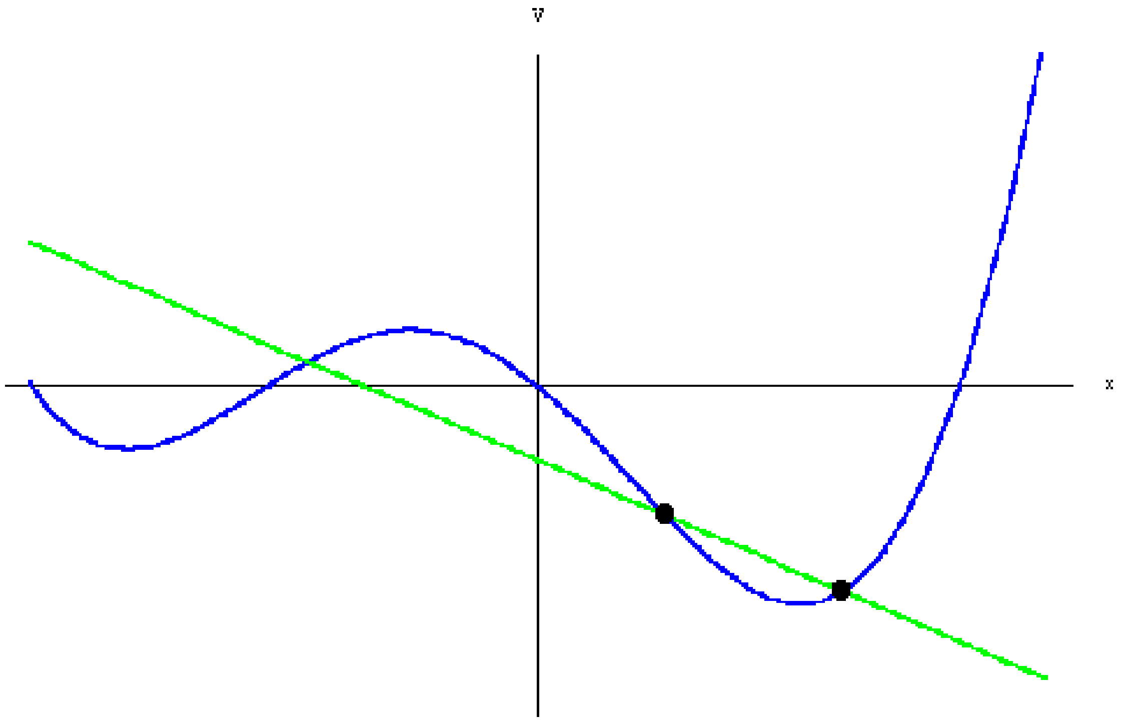
(a) The secant line through $(c, f(c))$ and $(c + \Delta x, f(c + \Delta x))$



As *Q* approaches *P*, the secant line approaches to the tangent line

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

EQUATION



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

STEPS

A. Purpose of a Tangent Line

1. The line shares a point with the curve in question.
2. At the shared point, the derivative of the curve is equal to the slope of the line.

B. Finding the Derivative by the Limit Process

1. Apply the Difference Quotient the equation
2. Add the limit to each step as the goal is to determine the limit when $h \to 0$; the difference is h. It approaches at zero because the slope of the graph of f at $x = c$.

Review

1) **Solve for $f(3)$ of $f(x) = x^2 - 3$** $f\left(3\right) = 6$

2) **Solve for $f(h)$ of $f(x) = x^2 - 3$** $f\left(h\right) = h^2 - 3$

3) **Solve for $f(x + \Delta x)$ of $f(x) = x^2 - 3$** $f\left(x+\Delta x\right) = x^2 + 2x\Delta x + \left(\Delta x\right)^2 - 3$

EXAMPLE 1

Use the limit process to find the derivative of $f(x) = 2x - 3$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{2(x+h) - 3 - (2x-3)}{h}$$

$$\lim_{h \to 0} \frac{\cancel{2x} + 2h \cancel{- 3} \cancel{- 2x} \cancel{+ 3}}{h}$$

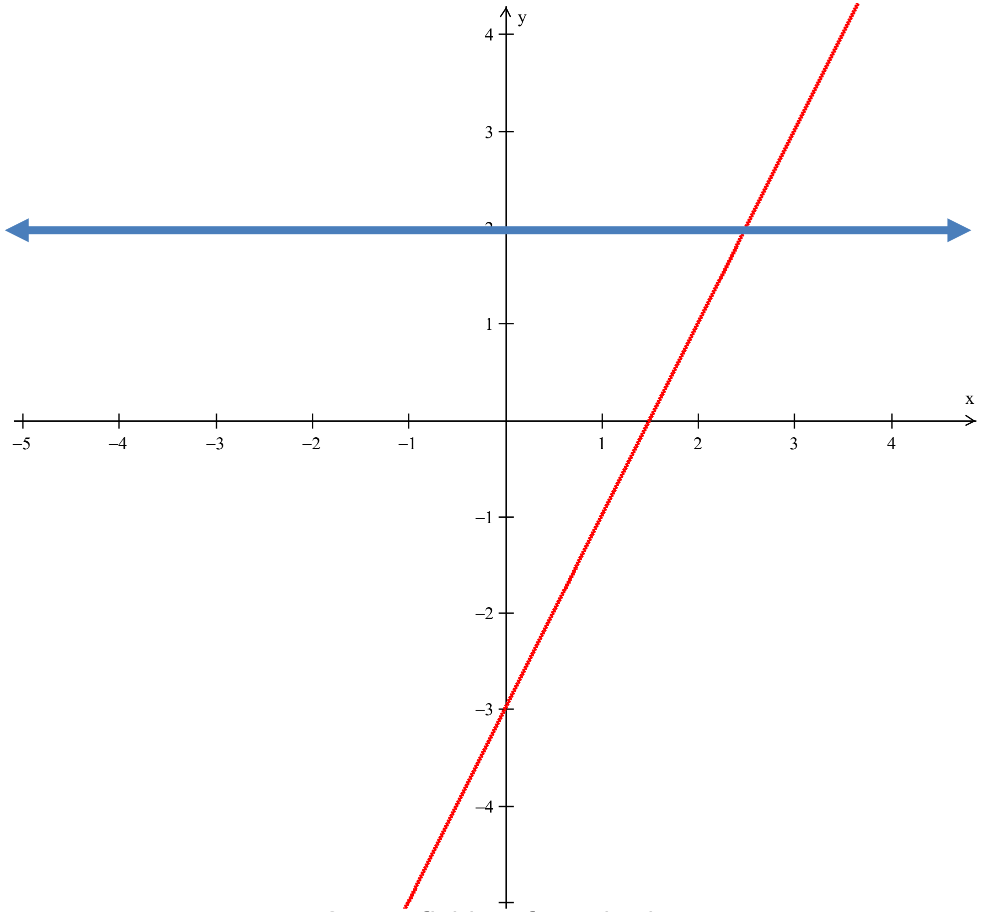
$$\lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}}$$

$$f'(x) = 2$$

What is the slope of this function?

Example 1

Use the limit process to find the derivative of $f(x) = 2x - 3$



Example 2

Use the limit process to find the derivative of $f(x) = 2x^2 - 16x + 35$

$$f'(x) = 4x - 16$$

YOUR TURN

Use the limit process to find the derivative of $f(x) = x^2 - 3$

$$f'(x) = 2x$$

Example 3

Use the limit process to find the derivative of $f(x) = \sqrt{5x - 8}$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{\sqrt{5(x+h) - 8} - \sqrt{5x - 8}}{h}$$

$$\lim_{h \to 0} \frac{\sqrt{5x + 5h - 8} - \sqrt{5x - 8}}{h}$$

Example 3

Use the limit process to find the derivative of $f(x) = \sqrt{5x - 8}$

$$\lim_{h \to 0} \frac{\sqrt{5x+5h-8} - \sqrt{5x-8}}{h}$$

$$\lim_{h \to 0} \left(\frac{\sqrt{5x+5h-8} - \sqrt{5x-8}}{h} \right) \left(\frac{\sqrt{5x+5h-8} + \sqrt{5x-8}}{\sqrt{5x+5h-8} + \sqrt{5x-8}} \right)$$

$$\lim_{h \to 0} \frac{\cancel{5x} + 5\cancel{h} \cancel{-8} \cancel{-5x} \cancel{+8}}{\cancel{h}\left(\sqrt{5x+5h-8} + \sqrt{5x-8}\right)}$$

$$\lim_{h \to 0} \frac{5}{\sqrt{5x+5h-8} + \sqrt{5x-8}}$$

EXAMPLE 3

Use the limit process to find the derivative of $f(x) = \sqrt{5x - 8}$

$$\lim_{h \to 0} \frac{5}{\sqrt{5x + 5h - 8} + \sqrt{5x - 8}}$$

$$\lim_{h \to 0} \frac{5}{\sqrt{5x + 5(0) - 8} + \sqrt{5x - 8}}$$

$$\frac{5}{\sqrt{5x - 8} + \sqrt{5x - 8}}$$

Example 3

Use the limit process to find the derivative of $f(x) = \sqrt{5x - 8}$

$$\frac{5}{\sqrt{5x-8}+\sqrt{5x-8}}$$

$$\frac{5}{2\sqrt{5x-8}}$$

$$\boxed{\frac{5}{2\sqrt{5x-8}}}$$

YOUR TURN

Use the limit process to find the derivative of $\boldsymbol{f(x) = \sqrt{x - 1}}$

$$\frac{1}{2\sqrt{x-1}}$$

Using the Limit Process to Find Tangent Line

A. Use the limit process to find the derivative
B. Plug in x of the derivative to determine the slope
C. Plug into Point-Slope form, $y - y_1 = m(x - x_1)$ with the given x and y coordinates, and slope
D. AP Hint: For free-response AP Questions, do not need to simplify point-slope form

Example 4

Find the slope of the graph of $f(x) = x^2 - 8x + 9$ at the point $(3, -6)$ using the limit process. Then, find the equation of the tangent line.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{(x+h)^2 - 8(x+h) + 9 - (x^2 - 8x + 9)}{h}$$

$$\lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 8x - 8h + 9 - x^2 + 8x - 9}{h}$$

Example 4

Find the slope of the graph of $f(x) = x^2 - 8x + 9$ at the point $(3, -6)$ using the limit process. Then, find the equation of the tangent line.

$$\lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 8x - 8h + 9 - x^2 + 8x - 9}{h}$$

$$\lim_{h \to 0} \frac{2xh + h^2 - 8h}{h}$$

$$\lim_{h \to 0} \frac{h\left(2x + h - 8\right)}{h}$$

Example 4

Find the slope of the graph of $f(x) = x^2 - 8x + 9$ at the point $(3, -6)$ using the limit process. Then, find the equation of the tangent line.

$$\lim_{h \to 0} \frac{h(2x + h - 8)}{h}$$

$$\lim_{h \to 0} (2x + h - 8)$$

$$\lim_{h \to 0} (2(3) + (0) - 8)$$

$$2x - 8$$

$$y + 6 = -2(x - 3)$$

EXAMPLE 5

Find the slope of the graph of $f(x) = \frac{3}{x}$ at the point $(3, 1)$ using the limit process. Then, find the equation of the tangent line.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h}$$

$$\lim_{h \to 0} \frac{3x - 3(x+h)}{x(x+h)} \cdot \frac{1}{h}$$

Example 5

Find the slope of the graph of $f(x) = \frac{3}{x}$ at the point $(3, 1)$ using the limit process. Then, find the equation of the tangent line.

$$\lim_{h \to 0} \frac{-3h}{x(x+h)} \cdot \frac{1}{h}$$

$$\lim_{h \to 0} -\frac{3}{x(x+h)}$$

$$f'(x) = -\frac{3}{x^2}$$

EXAMPLE 5

Find the slope of the graph of $f(x) = \frac{3}{x}$ at the point $(3, 1)$ using the limit process. Then, find the equation of the tangent line.

$$f'(x) = \frac{-3}{x^2}$$

$$f'(3) = \frac{-3}{(3)^2}$$

$$f'(3) = -\frac{1}{3}$$

$$y - y_1 = m(x - x_1)$$

EXAMPLE 5

Find the slope of the graph of $f(x) = \frac{3}{x}$ at the point $(3, 1)$ using the limit process. Then, find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$

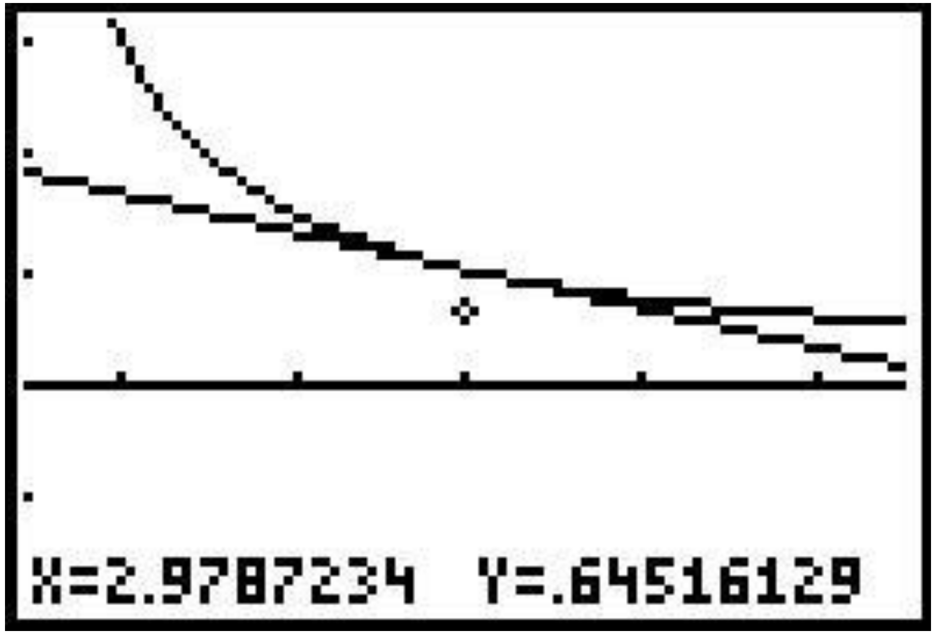
$$y - y_1 = -\frac{1}{3}(x - x_1)$$

$$y - (1) = -\frac{1}{3}(x - (3))$$

$$y - 1 = -\frac{1}{3}(x - 3)$$

Example 5

Find the slope of the graph of $f(x) = \frac{3}{x}$ at the point $(3, 1)$ using the limit process. Then, find the equation of the tangent line.



YOUR TURN

Find the slope of the graph of $f(x) = x^2 + 2x + 1$ at the point $(-3, 4)$ using the limit process. Then, find the equation of the tangent line.

$$y - 4 = -4(x + 3)$$

Example 6

Given the function $y = \frac{1}{x}$ anchored at the point $\left(2, \frac{1}{2}\right)$, find the slope of the secant line drawn through the point with x-coordinate $2 + h$. Use the expression to find the slope of the tangent to the graph of $y = \frac{1}{x}$ at the point $\left(2, \frac{1}{2}\right)$.

2 Questions:

What is a secant line? 2 points that make a line

What is a tangent line? 1 point that make a line

EXAMPLE 6

Given the function $y = \frac{1}{x}$ anchored at the point $\left(2, \frac{1}{2}\right)$, find the slope of the secant line drawn through the point with x-coordinate $2 + h$. Use the expression to find the slope of the tangent to the graph of $y = \frac{1}{x}$ at the point $\left(2, \frac{1}{2}\right)$.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{f(2+h+h) - f(2+h)}{h}$$

$$\lim_{h \to 0} \frac{\frac{1}{2+h+h} - \frac{1}{2+h}}{h}$$

EXAMPLE 6

Given the function $y = \frac{1}{x}$ anchored at the point $\left(2, \frac{1}{2}\right)$, find the slope of the secant line drawn through the point with x-coordinate $2 + h$. Use the expression to find the slope of the tangent to the graph of $y = \frac{1}{x}$ at the point $\left(2, \frac{1}{2}\right)$.

$$\lim_{h \to 0} \frac{\frac{1}{2+h+h} - \frac{1}{2+h}}{h}$$

$$\lim_{h \to 0} \frac{-h}{(2+2h)(2+h)} \cdot \frac{1}{h}$$

$$\lim_{h \to 0} \frac{-1}{(2+2h)(2+h)}$$

Slope of Tangent Line: $-\frac{1}{4}$

AP Multiple Choice Practice Question 1 (non-calculator)

If $f(2) = 3$ and $f'(2) = -1$, find an equation of the tangent line when $x = 2$

(A) $y + 1 = 2(x - 2)$

(B) $y - 3 = 2(x + 1)$

(C) $y - 2 = 3(x + 1)$

(D) $y - 3 = -1(x - 2)$

AP Multiple Choice Practice Question 1 (non-calculator)

If $f(2) = 3$ and $f'(2) = -1$, find an equation of the tangent line when $x = 2$

Vocabulary	Connections and Process	Answer and Justifications
Definition of a Derivative Function Notation	$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$ $y - y_1 = m(x - x_1)$ $m = f'(x)$ $y - y_1 = f'(x - x_1)$ $y - (3) = f'(x - 2)$ $y - (3) = (-1)(x - 2)$	D At $(2,3)$, the slope at the point is -1. Therefore, the equation of the tangent line is $y - 3 = -1(x - 2)$

ASSIGNMENT

Page 103

5-11 odd, 17, 21-29 odd