§1.4B: Intermediate Value Theorem, §1.5: Infinite Limits, and §3.5: Limits at Infinity
"I WILL…
…apply the Intermediate Value Theorem to real life situations."

I. Intermediate Value Theorem

A. If $f(x)$ is continuous on the closed interval $[a, b]$

B. $f(a) \neq f(b)$

C. If k is between $f(a)$ and $f(b)$ then there exists a number c between a and b for $f(c) = k$

Ex 1: If $f(x) = x^2 + x - 1$, prove the IVT holds through the indicated interval of $[0,5]$. If the IVT applies, find the value of c for $f(c) = 11$. What are the extremes? (other words $f(a)$ and $f(b)$)?	Ex 2: If $f(x) = x^2 - 6x + 8$, prove the IVT holds through the indicated interval of $[0,3]$. If the IVT applies, find the value of c for $f(c) = 3$.

Your Turn: If $f(x) = \frac{1}{x-2}$, use the Intermediate Value Theorem to prove there is zero on the interval $\left[\frac{5}{2}, 7\right]$ if $f(c) = \frac{1}{4}$.

II. To Earn Full Credit:

A. The function, $f(x)$ (or whatever they give) is identified and stated to be continuous.

B. Include the function is continuous in $(a,\ b)$ where a and b are defined

C. State the value of c, if asked to be defined.

III. Piecewise Functions

A. For a piecewise function to be continuous each function must be continuous on its specified interval and the limit of the endpoints of each interval must be equal.

Ex 3: What value of a will make the given piecewise function $f(x)$ continuous at $x = -3$ of $f(x) = \begin{cases} \frac{2x^2+5x-3}{x^2-9}, x \neq -3 \\ a, \qquad\qquad x = -3 \end{cases}$?

IV. Infinite Limits

A. Infinite Limits are limits in which $f(x)$ increases or decreases without bound as x approaches c or $\lim_{x \to c} f(x) = \pm\infty$, does not exist. The statement, "x approaches to infinity" really describes the limit to reach a constant number.

B. Let f and g be continuous on an open interval containing c. If $f(c) \neq\ 0$, $g(c)\ =\ 0$, then $h(x) = \frac{f(x)}{g(x)}$ has a vertical asymptote at $x = c$.

V. Steps

A. Simplify the rational function

B. Establish the limit from the left side and right side

C. If the limits are the same, it is the established limit.

D. If the limits are not the same, the limit fails to exist.

Review: Determine $\lim_{x \to 2^-} \frac{3}{x-2}$ and $\lim_{x \to 2^+} \frac{3}{x-2}$	Ex 4: Solve the vertical asymptotes (if any) of $f(x) = \frac{2}{\sin(x)}$ and $\lim_{x \to 0} = \frac{2}{\sin x}$

Your Turn: Solve the vertical asymptotes (if any) of $f(x) = \frac{1}{x^2}$ and $\lim_{x \to 0} = \frac{1}{x^2}$

VI. Properties of Infinite Limits

A. As we let $\lim_{x \to c} f(x) = \infty$ and $\lim_{x \to c} g(x) = L$:

B. Types:

1. Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] = \infty \pm L = \infty$
2. Product: $\lim_{x \to c} [f(x)g(x)] = \infty$ if $\lim_{x \to c} g(x)$ or $L > 0$

   $\lim_{x \to c} [f(x)g(x)] = -\infty$ if $L < 0$
3. Quotient: $\lim_{x \to c} \left[\frac{f(x)}{g(x)}\right] = \frac{\infty}{L} = \frac{BIGGER}{SMALLER} = \pm\infty$ (DNE) "It's BS so it is DNE"

   $\lim_{x \to c} \left[\frac{g(x)}{f(x)}\right] = \frac{L}{\infty} = \frac{SMALLER}{BIGGER} = 0$

C. Similar properties hold for one–sided limits and for functions for which the limit of $f(x)$ as x approaches c is $+\infty$.

Ex 5: If $\lim_{x \to 1^-} (x^2 + 1) = 2$ and $\lim_{x \to 1^-} \frac{1}{x-1} = -\infty$, solve $\lim_{x \to 1^-} \frac{x^2+1}{1/(x-1)}$.	Your Turn: If $\lim_{x \to 1^-} (x^2 + 1) = 2$ and $\lim_{x \to 1^-} (\cot \pi x) = -\infty$, solve $\lim_{x \to 1^-} \frac{x^2+1}{\cot \pi x}$.
Ex 6: Solve $\lim_{x \to 3^+} \frac{2}{x-3}$.	Your Turn: Solve $\lim_{x \to 1^+} \frac{x^2-3x}{x-1}$

VII. Steps

A. Establish the highest exponent
B. Divide each term by the biggest exponent of the denominator
    1. Bottom Exponent is Bigger: "0"; $x = 0$
    2. Top Exponent is Bigger: No (Simplify)
    3. Same Exponent on Top and Bottom: Divide the leading Coefficients
C. Simplify

Ex 7: Solve $\lim_{x\to-\infty} \frac{3x^2}{x^2+1}$ and $\lim_{x\to\infty} \frac{3x^2}{x^2+1}$	Ex 8: Solve $\lim_{x\to\infty} \frac{6\sqrt[3]{x}+3\sqrt[6]{x}+4}{\sqrt[3]{8x}-\sqrt[6]{x}-10}$
Your Turn: Solve $\lim_{x\to\infty} \left(\frac{x^2-1}{1-2x^2}\right)$	Ex 9: Solve $\lim_{x\to\infty} \frac{2x-1}{x+1}$
Ex 10: Solve $\lim_{x\to\infty} \frac{3x+5}{4x^2+1}$	Ex 11: Solve $\lim_{x\to-\infty} \frac{5x^4-2x^2}{4x^2+1}$
Your Turn: Solve $\lim_{x\to\infty} \frac{2x^2-3}{7x+4}$	

Identify the following of functions in order of "bigness" and how they grow, e^x , $\ln x$, c, x^2, $\sin x$ or $\cos x$:

Ex 12: Solve $\lim_{x\to\infty} \frac{e^x}{\ln x}$	Ex 13: Solve $\lim_{x\to\infty} \frac{\sqrt{3x^2+6}}{5-2x}$
Ex 14: Solve $\lim_{x\to-\infty} \frac{\sqrt{2x^2-1}}{-5x+3}$	Your Turn: Solve $\lim_{x\to-\infty} \frac{\sqrt{3x^2+3}}{x+3}$
Ex 15: Identify the horizontal asymptote of $g(x) = \frac{\sqrt{4x^2+5}}{3-2x}$	Ex 16: Identify the horizontal asymptote of $g(x) = \frac{\sqrt{e^{2x}+1}}{1+e^x}$

Ex 17: Identify the horizontal asymptotes of $f(x) = \frac{4+6e^x}{5-9e^x}$

AP 1) Let f be a continuous function on the closed interval $[-3,\ 6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that:

(A) $f'(c) = \frac{4}{9}$ for at least one c between -3 and 6

(B) $-1 \leq\ f(x) \leq 3$ for all x between -3 and 6

(C) $f(c) = 1$ for at least one c between -3 and 6

(D) $f(c) = 0$ for at least one c between -1 and 3

Vocabulary	Connections and Process	Answer and Justifications

AP 2) Solve $\lim_{t\to\infty} \frac{t^{4/3}+t^{1/3}}{\left(4t^{2/3}+1\right)^2}$

(A) $-\frac{1}{16}$ (B) $\frac{1}{4}$ (C) 0 (D) $\frac{1}{16}$

Vocabulary	Connections and Process	Answer and Justifications

Assignment: Worksheet