Calculus AP/Dual Worksheet Rev 2017 **Name:** ______________________________
§1.4B: Intermediate Value Theorem **Date:** ____________________ **Period** ______

Sketch the graph of a function f that satisfies the stated conditions.

1) f has a limit at $x = 3$, but it is not continuous at $x = 3$.

2) f is not continuous at $x = 3$, but if its value at $x = 3$ is changed from $f(3)=1$ to $f(3)=0$, f becomes continuous at $x = 3$.

3) f has a removable discontinuity at $x = c$ for which $f(c)$ is undefined.

4) f has a removable discontinuity at $x = c$ for which $f(c)$ is defined.

5) Use the definition of continuity to prove $f(x)=\frac{x^2-5x+4}{x-1}$ where $c=1$ and that the function is discontinuous at the given value of c. Sketch the graph of the function.

Use the definition of continuity to find the values of k and/or m that will make the function continuous everywhere.

6) $f(x)=\begin{cases} kx^2, & x\le 2 \\ 2x+k, & x>2 \end{cases}$

7) $g(x)=\begin{cases} x^2+5, & x>2 \\ m(x+3)+k, & -1<x\le 2 \\ 2x^3+x+7, & x\le -1 \end{cases}$

Function f and a closed interval $[a,b]$ are given. Show whether the conditions of the **Intermediate Value Theorem** hold for the given value of k. If the conditions hold, find a number c such that $f(c)=k$. If the theorem does not hold, give the reason.

8) $f(x)=2+x+x^2$
$[a,b]=[0,3]$
$f(c)=1$

9) $f(x)=\sqrt{25-x^2}$
$[a,b]=[-4.5,\ 3]$
$k=3$

10) $f(x)=\frac{1}{x-2}$
$[a,b]=[3,\ 5]$
$k=\frac{5}{6}$

11) Use the Intermediate Value Theorem to show that $x^3+x=0$ has a root in the $I[-1,2]$.

12) Use the Intermediate Value Theorem to show that $\cos x = x$ has a root in the $I\left[0,\frac{\pi}{2}\right]$.

13) One night in January, the outside temperature at midnight was 42°F. At 10 AM the next morning, the temperature was 57°F.

(a) Must there have been a time between midnight and 10 AM when the temperature was 50°F? Explain how you know.

(b) Must there have been a time between midnight and 10 AM when the temperature was 40°F? Explain how you know.

(c) Could there have been a time between midnight and 10 AM when the temperature was 40°F? Explain how you know.

14) Consider the function, $f(x) = x^k - 3x^2 + 1$ where k is a non-zero constant. Let $k = 3$ so that $f(x) = x^k - 3x^2 + 1$. Explain why there must be a value r for $2 < r < 4$ such that $f(r) = 0$.

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t days	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

15) The twice-differential function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$sampled at various times during the time interval $0 \leq t \leq 30$ days. At time $t = 30$, the reservoir contains 125 GL of water.

Explain why there must be at least one time t, other than $t = 10$, such that $W'(t) = 0.7\ GL/day$.